The Truth About Acceptance Sampling: Part One

What can you say about this lot?

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One of the common tools of Quality Assurance is Acceptance Sampling. Acceptance Sampling uses the observed properties of a sample drawn from a lot or batch to make a decision about whether to accept or reject that lot or batch. While the textbooks are full of complex descriptions of various Acceptance Sampling plans, there are some very important aspects of Acceptance Sampling that are not included in the textbooks. These are the topic of this paper.

EXTRAPOLATION

In the interest of simplicity, the product that has been measured will be referred to as the "sample," while the product that has not been measured will be referred to as the "lot." Every time that we attempt to use a sample to characterize the conformity of a lot we will be making an extrapolation from the product that has been measured to the product that has not been measured. Now extrapolation is beyond the scope of most courses of instruction and education simply because it is so complex. Yet, typically, it is the one thing that we are called on to do on a daily basis. So how can we extrapolate from the sample to the lot?

Extrapolation inevitably requires an assumption that the sample is *representative* of the lot. When this assumption is appropriate, the extrapolation will make sense. When this assumption is wrong, the extrapolation will also be wrong. No matter what computations we may use, the validity of our statements about the lot will always come back to the question of representativeness. The ability of any sample to represent a lot will be affected by the homogeneity of the lot itself and the manner in which the sample is selected from the lot.

We shall begin with the case of random samples drawn from uniform lots. Given a lot having uniform quality, and given a random sample obtained from this lot, we can use probability theory to develop a mathematical treatment of the uncertainty of the extrapolation from the sample to the lot.

ESTIMATES OF LOT QUALITY

Needless to say, the twin assumptions of uniform lot quality and a random sample remove a host of problems. We will discuss the assumptions more completely in Part Two. A random sample may be broadly defined as one that is obtained in such a way that every item in the lot has the same chance of being included in the sample. When these two assumptions are generally satisfied, the following results will hold.

Whether we are measuring a property, testing for functionality, or counting blemished items, a sample of $\mathbf{n}$ items will contain some fixed number of nonconforming items, Y, and the sample fraction nonconforming will be:

$$\hat{p} = \frac{Y}{\mathbf{n}}$$

This value is known as the binomial point estimate for the lot fraction nonconforming. For hundreds of years it has been known to be the best single-number estimate of the lot fraction nonconforming. However, we need more than a single number estimate. We also need an interval estimate in order to incorporate the uncertainty of our extrapolation from the sample to the lot.

The best modern practice here is to use a 95% Agresti-Coull interval estimate. Here we begin with a biased point estimate that is found by first "adding two successes and two failures" to the values for Y and $\mathbf{n}$ and computing:

$$\tilde{p} = \frac{Y+2}{\mathbf{n}+4}$$

Next we use this biased point estimate to compute the 95% Agresti-Coull Interval Estimate according to the formula:

$$\tilde{p} \pm 1.96 \sqrt{\frac{\tilde{p}\,(1-\tilde{p})}{\mathbf{n}+4}}$$

(In my article "Estimating the Fraction Nonconforming" ***QDD***, June, 2011, I mislabeled this formula by calling it the "Wilson interval estimate.") Unlike the more common Wald interval estimate, the formula above is good all the way down to $Y = 0$ and all the way up to $Y = \mathbf{n}$. When this formula results in a lower end point that falls below zero, use zero instead of the computed value. When it results in an upper end point that falls above one, use one instead of the computed value. A table of 95% Agresti-Coull interval estimates for various values of Y and $\mathbf{n}$ is given in Figure 1.

For the situation where $\mathbf{n}$ exceeds ten percent of the lot size, N, the 95% Agresti-Coull interval estimate formula above must be modified to include a finite population correction factor:

$$\tilde{p} \pm 1.96 \sqrt{1-\frac{\mathbf{n}}{N}} \sqrt{\frac{\tilde{p}\,(1-\tilde{p})}{\mathbf{n}+4}}$$

This formula will tighten up the end points to take into account the depletion of the lot by the sample.

	***Y* = 0**		***Y* = 1**		***Y* = 2**		***Y* = 3**		***Y* = 4**		***Y* = 5**		***Y* = 6**	
n	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
5	0.0	49.4	2.5	64.1	12.0	76.9	23.1	88.0	35.9	97.5	50.6	100.		
10	0.0	32.6	0.0	42.9	4.9	52.2	10.6	60.8	16.9	68.8	23.8	76.2	31.2	83.1
15	0.0	24.3	0.0	32.2	2.7	39.4	6.5	46.1	10.7	52.5	15.2	58.5	19.9	64.3
20	0.0	19.4	0.0	25.7	1.8	31.6	4.6	37.1	7.7	42.3	11.0	47.4	14.5	52.2
25	0.0	16.1	0.0	21.4	1.2	26.3	3.5	31.0	5.9	35.4	8.6	39.7	11.3	43.9
30	0.0	13.8	0.0	18.4	0.9	22.6	2.8	26.6	4.8	30.5	7.0	34.2	9.3	37.8
35	0.0	12.1	0.0	16.1	0.7	19.8	2.3	23.3	4.1	26.7	5.9	30.0	7.8	33.2
40	0.0	10.7	0.0	14.3	0.6	17.6	2.0	20.7	3.5	23.8	5.1	26.7	6.8	29.6
45	0.0	9.6	0.0	12.8	0.5	15.8	1.7	18.7	3.1	21.4	4.5	24.1	6.0	26.7
50	0.0	8.7	0.0	11.7	0.4	14.4	1.5	17.0	2.7	19.5	4.0	21.9	5.3	24.3
60	0.0	7.4	0.0	9.9	0.3	12.2	1.2	14.4	2.2	16.5	3.3	18.6	4.4	20.6
70	0.0	6.4	0.0	8.5	0.3	10.6	1.0	12.5	1.9	14.3	2.8	16.1	3.7	17.9
80	0.0	5.6	0.0	7.5	0.2	9.3	0.9	11.0	1.6	12.7	2.4	14.2	3.2	15.8
100	0.0	4.6	0.0	6.1	0.2	7.5	0.7	8.9	1.3	10.3	1.9	11.5	2.6	12.8
120	0.0	3.8	0.0	5.1	0.1	6.3	0.6	7.5	1.1	8.6	1.6	9.7	2.1	10.8
150	0.0	3.1	0.0	4.1	0.1	5.1	0.4	6.0	0.8	7.0	1.3	7.8	1.7	8.7
175	0.0	2.7	0.0	3.6	0.1	4.4	0.4	5.2	0.7	6.0	1.1	6.8	1.4	7.5
200	0.0	2.3	0.0	3.1	0.1	3.9	0.3	4.6	0.6	5.3	0.9	5.9	1.3	6.6
250	0.0	1.9	0.0	2.5	0.0	3.1	0.3	3.7	0.5	4.2	0.7	4.8	1.0	5.3
300	0.0	1.6	0.0	2.1	0.0	2.6	0.2	3.1	0.4	3.5	0.6	4.0	0.8	4.4

	***Y* = 7**		***Y* = 8**		***Y* = 9**		***Y* = 10**		***Y* = 11**		***Y* = 12**		***Y* = 13**	
n	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
10	39.2	89.4	47.8	95.1	57.1	100.	67.4	100.						
15	24.9	69.8	30.2	75.1	35.7	80.1	41.5	84.8	47.5	89.3	53.9	93.5	60.6	97.3
20	18.1	56.9	21.9	61.4	25.9	65.8	30.0	70.0	34.2	74.1	38.6	78.1	43.1	81.9
25	14.2	47.9	17.2	51.8	20.3	55.6	23.5	59.3	26.7	62.9	30.1	66.5	33.5	69.9
30	11.6	41.3	14.1	44.7	16.6	48.1	19.2	51.4	21.9	54.6	24.6	57.7	27.4	60.8
35	9.9	36.3	11.9	39.3	14.1	42.3	16.3	45.3	18.5	48.1	20.8	51.0	23.2	53.7
40	8.5	32.4	10.3	35.1	12.2	37.8	14.1	40.4	16.1	43.0	18.1	45.6	20.1	48.1
45	7.5	29.2	9.1	31.7	10.8	34.1	12.4	36.5	14.2	38.9	15.9	41.2	17.7	43.5
50	6.7	26.6	8.2	28.9	9.6	31.1	11.1	33.3	12.7	35.5	14.2	37.6	15.8	39.7
60	5.5	22.6	6.7	24.5	7.9	26.4	9.2	28.3	10.5	30.2	11.7	32.0	13.1	33.8
70	4.7	19.6	5.7	21.3	6.8	23.0	7.8	24.6	8.9	26.2	10.0	27.8	11.1	29.4
80	4.1	17.3	5.0	18.8	5.9	20.3	6.8	21.8	7.7	23.2	8.7	24.6	9.7	26.0
100	3.3	14.1	3.9	15.3	4.7	16.5	5.4	17.7	6.1	18.9	6.9	20.0	7.7	21.2
120	2.7	11.8	3.3	12.9	3.9	13.9	4.5	14.9	5.1	15.9	5.7	16.9	6.4	17.8
150	2.1	9.5	2.6	10.4	3.1	11.2	3.6	12.0	4.1	12.8	4.6	13.6	5.1	14.4
175	1.8	8.2	2.2	9.0	2.6	9.7	3.0	10.4	3.5	11.1	3.9	11.8	4.3	12.4
200	1.6	7.2	1.9	7.9	2.3	8.5	2.7	9.1	3.0	9.7	3.4	10.3	3.8	10.9
250	1.3	5.8	1.5	6.3	1.8	6.8	2.1	7.3	2.4	7.8	2.7	8.3	3.0	8.8
300	1.1	4.9	1.3	5.3	1.5	5.7	1.8	6.1	2.0	6.6	2.2	7.0	2.5	7.4

	***Y* = 14**		***Y* = 15**		***Y* = 16**		***Y* = 17**		***Y* = 18**		***Y* = 19**		***Y* = 20**	
n	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
15	67.8	100.	75.7	100.										
20	47.8	85.5	52.6	89.0	57.7	92.3	62.9	95.4	68.4	98.2	74.3	100.	80.6	100.
25	37.1	73.3	40.7	76.5	44.4	79.7	48.2	82.8	52.1	85.8	56.1	88.7	60.3	91.4
30	30.3	63.8	33.2	66.8	36.2	69.7	39.2	72.6	42.3	75.4	45.4	78.1	48.6	80.8
35	25.6	56.5	28.0	59.2	30.5	61.8	33.0	64.4	35.6	67.0	38.2	69.5	40.8	72.0
40	22.1	50.6	24.2	53.0	26.4	55.4	28.5	57.8	30.7	60.2	33.0	62.5	35.2	64.8
45	19.5	45.8	21.4	48.0	23.2	50.2	25.1	52.4	27.1	54.6	29.0	56.7	31.0	58.8
50	17.5	41.8	19.1	43.9	20.8	45.9	22.4	47.9	24.2	49.9	25.9	51.9	27.6	53.8
60	14.4	35.6	15.7	37.4	17.1	39.1	18.5	40.9	19.9	42.6	21.3	44.3	22.7	46.0
70	12.2	31.0	13.4	32.6	14.5	34.1	15.7	35.6	16.9	37.1	18.1	38.7	19.3	40.1
80	10.7	27.4	11.6	28.8	12.7	30.2	13.7	31.6	14.7	32.9	15.7	34.3	16.8	35.6
100	8.5	22.3	9.2	23.5	10.0	24.6	10.8	25.7	11.7	26.8	12.5	27.9	13.3	29.0
120	7.0	18.8	7.7	19.8	8.3	20.7	9.0	21.7	9.7	22.6	10.3	23.5	11.0	24.5
150	5.6	15.2	6.1	16.0	6.6	16.8	7.1	17.5	7.7	18.3	8.2	19.1	8.8	19.8
175	4.8	13.1	5.2	13.8	5.7	14.5	6.1	15.1	6.6	15.8	7.0	16.4	7.5	17.1
200	4.2	11.5	4.5	12.1	4.9	12.7	5.3	13.3	5.7	13.9	6.1	14.5	6.5	15.0
250	3.3	9.3	3.6	9.8	3.9	10.2	4.2	10.7	4.6	11.2	4.9	11.7	5.2	12.1
300	2.8	7.8	3.0	8.2	3.3	8.6	3.5	9.0	3.8	9.4	4.1	9.8	4.3	10.1

Figure 1: 95% Agresti-Coull Interval Estimates for Percent Nonconforming in Lot

Say, for example, that a random sample of **n** = 250 parts has been selected from a lot containing 5000 parts, and when measured we find that $Y = 7$ of these parts are nonconforming. Then our best point estimate for the lot fraction nonconforming would be (7/250) = 2.8 percent, while from Table 1 we find our 95% Agresti-Coull interval estimate for the lot fraction nonconforming to be 1.3 percent to 5.8 percent. This means that we can be reasonably certain that the lot fraction nonconforming is no more than 5.8%, and also that it is probably greater than 1.3%. With these data we cannot really narrow it down more than this. If knowing that there is more than 1.3% nonconforming in this lot is bad enough, then reject this lot. On the other hand, if knowing that there is less than 5.8% nonconforming in this lot is good enough, then accept this lot. If 1.3% nonconforming is not bad enough to reject the lot or if 5.8% nonconforming is not good enough to accept the lot, *then you do not have sufficient evidence to take either action*. This is the truth about Acceptance Sampling. Think about this very carefully.

By forcing us to either accept or reject each lot, Acceptance Sampling leads you to take actions that are not always justified and which may be inappropriate. The interval estimate tells you what you know and also what you do not know.

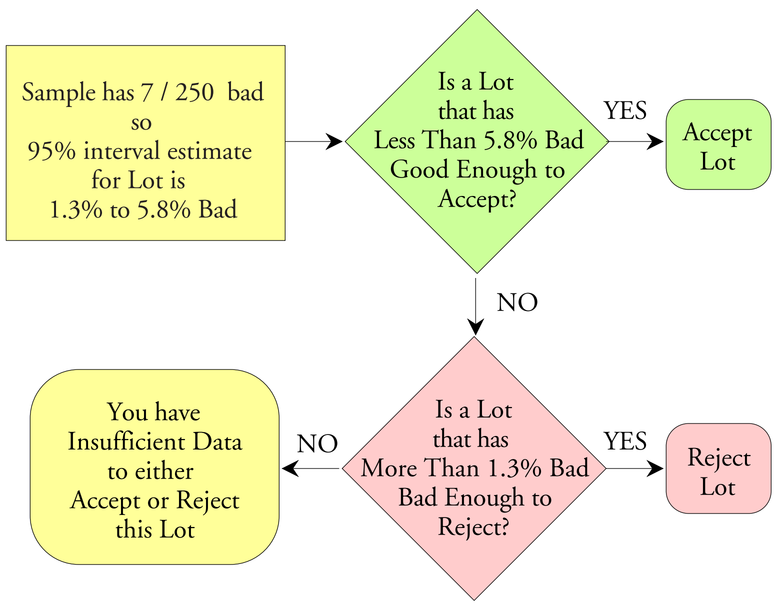

Figure 2: The Truth About Acceptance Sampling

SUMMARY

The interval estimates above tell you what you know about the lot based on the sample. They are the only way to characterize each particular lot. All of the other numbers and computations associated with Acceptance Sampling tell you *nothing* about the lot you have just sampled. Instead, they attempt to characterize the quality in the warehouse, which will be the topic for Part Two in August.