Honest Precision to Tolerance Ratios

How to make sense of P/T ratios

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The precision to tolerance ratio is commonly used to characterize the usefulness of a measurement system. While this ratio is appealingly simple, it overstates the damage due to measurement error. In this paper we show how to compute *honest precision to tolerance ratios* that correctly describe each of three different guard-banded sets of manufacturing specifications. By presenting these options, rather than using single ratio and artificial guidelines to condemn the measurement process, this approach provides flexibility based on knowledge.

THE PRECISION TO TOLERANCE RATIO

The P/T ratio was originally created in an attempt to describe how much of the specified tolerance was "consumed" by measurement error. While this ratio is quite simple, it fails to do what was intended. Originally the precision to tolerance ratio was defined using a numerator of 5.15 times the standard deviation of measurement error [or 5.15 sigma(e)]. The multiplier of 5.15 was obtained from the width of the interval that covers the middle 99 percent of a standard normal distribution. Thus the idea was to filter out 99 percent of the contribution of measurement error. Around 1990 the AIAG changed the multiplier from 5.15 to 6.00, and this is the version of the P/T ratio found in most software today.

$$\text{Precision to Tolerance Ratio} = P/T = \frac{6 \text{ sigma(e)}}{\text{specified tolerance}}$$

For an example consider a physical dimension that has specifications of 3.006 inches to 3.012 inches. Say that the measurement device records this dimension to the nearest 0.001 inch so that the measurement increment (MI) is one-thousandth of an inch.

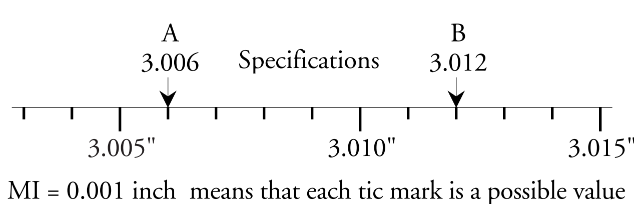

Figure 1: Specifications are usually expressed as possible values

Then, if measurement error is sigma(e) = 0.0007 inches the P/T ratio will be 0.6000 or sixty percent, which is a very bad number indeed. (Don't write to us if your computer gives you something different—it is probably not programmed to correctly compute the specified tolerance. For more on this see Figure 2.)

Regardless of whether we use either 5.15 or 6.0 in the numerator, the P/T ratio will overstate the impact of measurement error. This happens because neither version of this ratio has the appropriate mathematical foundation. As a result, neither version computes the "consumption" in the proper manner. Since guard-bands based on the traditional P/T ratio will end up rejecting an excess amount of good product, all producers should be interested in computing appropriate manufacturing specifications.

GUARD BANDS AND MANUFACTURING SPECIFICATIONS

Like the P/T ratio, most guard-bands are based on a probability of getting a conforming measurement (an outcome) given that the item is nonconforming (a state of nature). While conditional probabilities of an outcome given a state of nature are inputs to the problem, they are not the results needed to properly construct guard-bands. What is needed are the *a posteriori* probabilities—the probability of a state of nature given an outcome (the probability of a conforming item given that the measurement falls within the manufacturing specifications). This necessity of reversing the form of the probabilities to answer practical questions is one of the fundamental laws of probability theory that has been known since the Eighteenth Century.

The mathematics behind computing the *a posteriori* probabilities needed for defining appropriate guard-bands for manufacturing specifications are outlined in Wheeler's article "Where do Manufacturing Specifications Come From?" ***Quality Digest Daily*** July 6, 2010, and they are given in greater detail in Chapter 14 of his book "***EMP III: Evaluating the Measurement Process and Using Imperfect Data***." These guard-bands are characterized by the minimum probability that the product is conforming (a state of nature) when the measurement falls within the manufacturing specifications (an observed outcome).

We shall use this approach to discuss three different sets of manufacturing specifications. For these three sets of guard-bands the Honest P/T ratios will be respectively, 22.5 percent, 45 percent, and 67.5 percent as large as the traditional P/T ratio.

85% MANUFACTURING SPECIFICATIONS

If you want the probability of conforming product to be *at least* 85 percent, then you will need to tighten the watershed specifications by guard-bands of GB(85%) measurement units on each end where:

GB(85%) = [0.675 sigma(e)] measurement units

If A and B denote the smallest and largest acceptable values, and if MI represents the measurement increment, the watershed specifications will be:

$$\text{Lower Watershed Specification} = A - 0.5 * MI$$
$$\text{Upper Watershed Specification} = B + 0.5 * MI$$

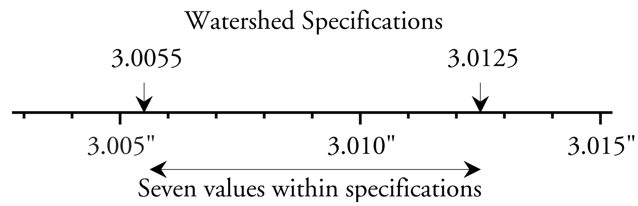

Figure 2: Watershed Specifications Fall Between the Possible Values

The 85% manufacturing specifications will be:

$$\text{Lower 85\% Manufacturing Specification} = A - 0.5 * MI + GB(85\%)$$
$$\text{Upper 85\% Manufacturing Specification} = B + 0.5 * MI - GB(85\%)$$

Items with measurement values that fall within these 85% manufacturing specifications will have at least an 85 percent likelihood of conforming to the customer specifications.

The proportion of the specified tolerance that is consumed by these 85 percent guard-bands is characterized by the Honest P/T (85%) ratio:

$$\text{Honest P/T (85\%)} = \frac{2 * GB(85\%)}{B - A + MI}$$

This Honest P/T (85%) ratio will be 22.5 percent as large as the original P/T ratio (when that value has been computed correctly using the watershed specifications).

For our example, A = 3.006″, B = 3.012″, MI = 0.001″, sigma(e) = 0.0007″, and the traditional P/T ratio is 0.60. The guard-band for 85% manufacturing specifications would be:

$$GB(85\%) = [\ 0.675 \ \text{sigma(e)}\ ] = 0.00047 \text{ inches}$$

So the 85% manufacturing specifications are:

$$A - 0.5 * MI + GB(85\%) = 3.00597''$$
$$B + 0.5 * MI - GB(85\%) = 3.01203''$$

And the possible values within these specifications are 3.006″ to 3.012″. When a measurement falls within this range the item has at least an 85% chance of being conforming. Here the Honest P/T (85%) ratio is 0.134, which describes that proportion of the specified tolerance (13.4%) that is consumed by the 85% guard-bands.

96% MANUFACTURING SPECIFICATIONS

If you want the probability of conforming product to be *at least* 96% then you will need to tighten the specifications by guard-bands of GB(96%) measurement units on each end where:

$$GB(96\%) = [\, 1.35 * \text{sigma(e)} \,] \text{ measurement units}$$

and the 96% manufacturing specifications will be:

$$\text{Lower 96\% Manufacturing Specification} = A - 0.5 * MI + GB(96\%)$$
$$\text{Upper 96\% Manufacturing Specification} = B + 0.5 * MI - GB(96\%)$$

Items with measurement values that fall within these 96% manufacturing specifications will have at least an 96 percent likelihood of conforming to the customer specifications.

The proportion of the specified tolerance that is consumed by these 96 percent guard-bands is characterized by the Honest P/T (96%) ratio:

$$\text{Honest P/T (96\%)} = \frac{2 * GB(96\%)}{B - A + MI}$$

This Honest P/T (96%) ratio will be 45 percent as large as the original P/T ratio.

For our example, A = 3.006″, B = 3.012″, MI = 0.001″, sigma(e) = 0.0007″, and the traditional P/T ratio is 0.60. The guard-bands for 96% manufacturing specifications would be:

$$GB(96\%) = 1.35 * \text{sigma(e)} = 0.00095 \text{ inches}$$

Here our 96% manufacturing specifications are:

$$A - 0.5 * MI + GB(96\%) = 3.00645''$$
$$B + 0.5 * MI - GB(96\%) = 3.01155''$$

And the possible values that fall within these specs are 3.007″ to 3.011″. When an item gets a measurement within this range it has at least an 96% chance of being conforming. Here the Honest P/T (96%) ratio is 0.270, which describes that proportion of the specified tolerance that is consumed by the 96% guard-bands.

99% MANUFACTURING SPECIFICATIONS

If you want the probability of conforming product to be *at least* 99% then you will need to tighten the specifications by guard-bands of GB(99%) measurement units on each end where:

$$GB(99\%) = [\, 2.025 * \text{sigma(e)} \,] \text{ measurement units}$$

and the 99% manufacturing specifications will be:

$$\text{Lower 99\% Manufacturing Specification} = A - 0.5 * MI + GB(99\%)$$

$$\text{Upper 99\% Manufacturing Specification} = B + 0.5 * MI - GB(99\%)$$

Items with measurement values that fall within these 99% manufacturing specifications will have at least an 99 percent likelihood of conforming to the customer specifications.

The proportion of the specified tolerance that is consumed by these 99 percent guard-bands is characterized by the Honest P/T (99%) ratio:

$$\text{Honest P/T (99\%)} = \frac{2 * GB(99\%)}{B - A + MI}$$

This Honest P/T (99%) ratio will be 67.5 percent as large as the original P/T ratio.

For our example, A = 3.006″, B = 3.012″, MI = 0.001″, sigma(e) = 0.0007″, and the traditional P/T ratio is 0.60. The guard-bands for 99% manufacturing specifications would be:

$$GB(99\%) = 2.025 * \text{sigma(e)} = 0.00142 \text{ inches}$$

Here our 99% manufacturing specifications are:

$$A - 0.5 * MI + GB(99\%) = 3.00692''$$
$$B + 0.5 * MI - GB(99\%) = 3.01108''$$

And the possible values that fall within these specs are 3.007″ to 3.011″. (Thus, due to the round-off of the measurement increment, in this case the 96% and 99% manufacturing specifications turn out to include the same range of possible values.) When an item gets a measurement within this range it has at least an 99% chance of being conforming. The Honest P/T (99%) ratio is 0.405, which describes that proportion of the specified tolerance that is consumed by the 99% guard-bands.

SUMMARY

This rigorous approach to the definition of guard-bands gives you a choice between three different sets of manufacturing specifications having different minimum probabilities of conforming product of 85 percent, 96 percent, or 99 percent. In this case the guard-bands for these manufacturing specifications consume 13.5 percent, 27 percent, and 40.5 percent of the specified tolerance respectively. The traditional P/T ratio claims that measurement error consumes 60 percent of the specified tolerance. Thus, the inflation that is inherent in traditional P/T ratio, plus the conservative nature of the traditional guideline for interpreting the P/T ratio, combine to effectively condemn most measurement systems. On the other hand, correctly computed guard-bands and their Honest P/T ratios give us options for using the current measurement system. Since we always have to work with imperfect measurement systems these options are important.

If we condemn a measurement system, then that system will have to be replaced. This will require time, effort, and capital expenditures. Since expenditures on measurement systems are always an overhead expense, they should not be undertaken unnecessarily.

If we do not condemn the measurement system, but use guard-bands based on the original P/T ratio, then the tightened specifications will be narrower than they should be and an excess amount of good product will be rejected unnecessarily.

The use of inflated P/T ratios and arbitrarily conservative guidelines may allow us to beat our vendors over the head, but it does nothing to increase quality, productivity, or competitive position. Understanding how to work within the limitations of the current measurement system allows us to avoid unnecessary costs.