Tightened 100% Inspection

Guard-bands may cost more than they help

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Inspection sounds simple. Screen out the bad stuff and ship the good stuff. However, measurement error will always create problems of misclassification where good stuff is rejected and bad stuff gets shipped. While guard-bands and tightened inspection have been offered as a way to remedy the problem of shipping bad stuff, it turns out that they are often prohibitively expensive in practice. Here we look at how tightened inspection improves the quality of the product stream and compare those improvements with the associated excess costs.

The Problem of Inspection

A product measurement, X, may be thought of as consisting of the product value, Y, plus some measurement error, E, so that $X = Y + E$. With this model, the relationship between X and Y can be shown using a bivariate normal probability model where:

$$Mean(X) = Mean(Y)$$

$$Var(X) = Var(Y) + Var(E)$$

$$Bivariate\ Correlation(X,Y) = \sqrt{ICC}$$

$$\text{where} \quad ICC = \frac{Var(Y)}{Var(X)} = 1 - \frac{Var(E)}{Var(X)}$$

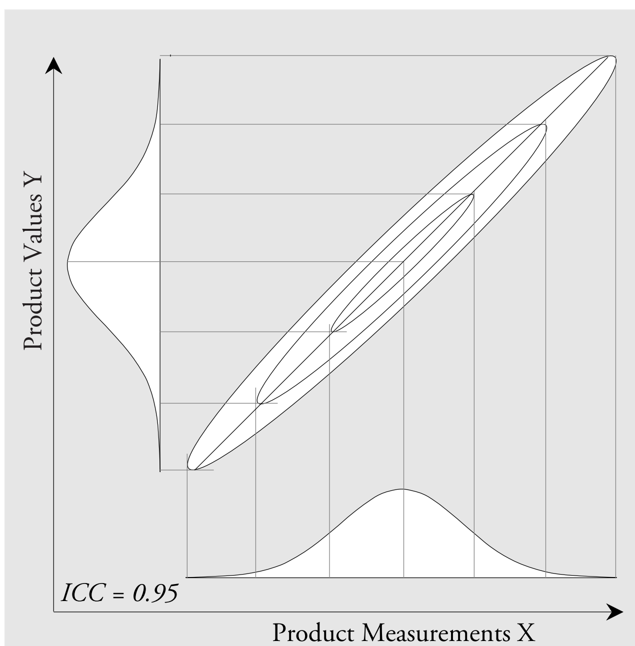

Figure 1: Bivariate Normal when *ICC* is 0.95

(The intraclass correlation coefficient (*ICC*) is a characterization of the relative utility of the measurement system for use with the product measurements.) This bivariate normal density function can be shown by contour ellipses in the x y plane. As the intraclass correlation decreases these ellipses get fatter. These ellipses completely and fully describe how the values for X vary with the values for Y, which is the essence of the inspection problem.

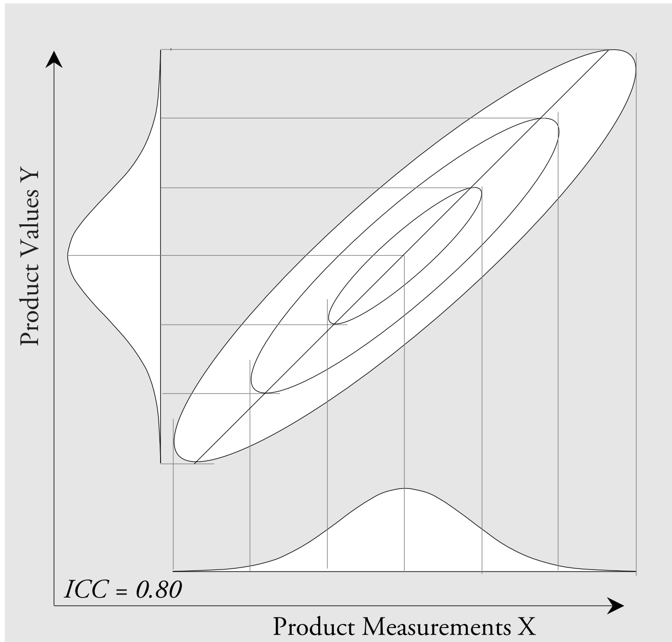

Figure 2: Bivariate Normal when *ICC* is 0.80

Here we are interested in those values for the product measurements X that correspond to the conforming items, Y. In Figure 3 the white region within the ellipse corresponds to those (x,y) coordinates that coincide with the conforming items.

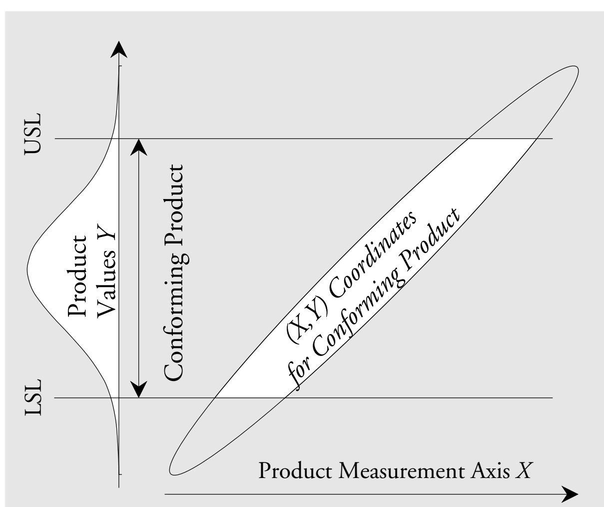

Figure 3: The *(x,y)* Coordinates for Conforming Product

In contrast to Figure 3, Figure 4 shows the (x,y) coordinates that correspond to the product that gets shipped following the 100% inspection.

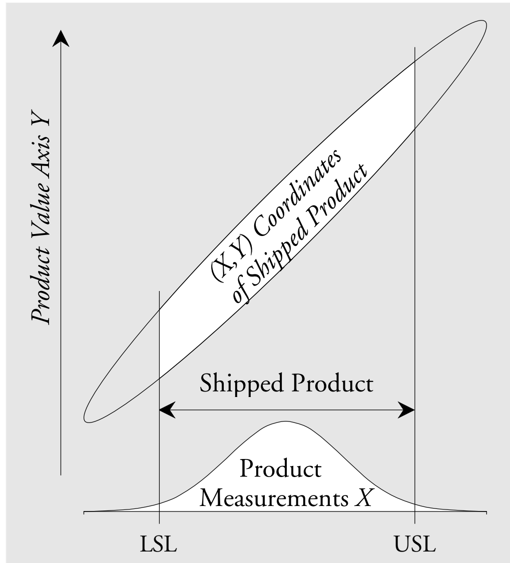

Figure 4: The *(x,y)* Coordinates for Product That Gets Shipped

Thus, the region of conforming product is not the same as the region of product that gets shipped. Moreover, the difference between these two regions increases as the ellipses get fatter. This difference is the essence of the problem of inspection. Using "good" as shorthand for conforming and "bad" as shorthand for nonconforming, we end up with four distinct outcomes:

- Good product that gets shipped (GS);
- good product that gets rejected (GR);
- bad product that gets shipped (BS); and
- bad product that gets rejected (BR).

Of course, good product that gets shipped (GS) and bad product that gets rejected (BR) are the desired outcomes for the inspection procedure. The good product that is rejected (GR) is a problem for the producer and the bad product that gets shipped (BS) is a problem for the customer. Figure 5 shows two regions where good product will be rejected (GR1 and GR2), and two regions where bad product will be shipped (BS1 and BS2).

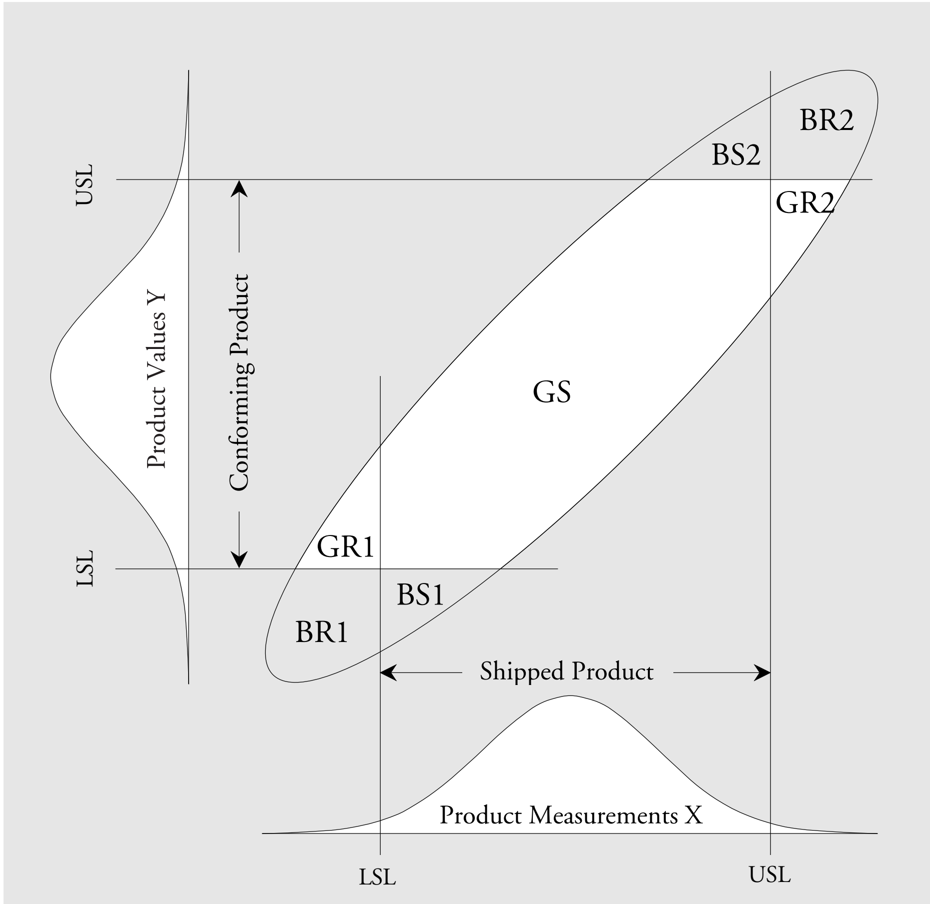

Figure 5: The Four Outcomes of 100% Inspection

By integrating the appropriate bivariate normal density functions over the seven regions shown we can determine how various measurement systems will work with various process capabilities. This was done for the combinations of sixteen different process capabilities with eighteen different measurement systems (different *ICC* values). For each of these 288 situations I looked at the total number of parts per thousand in the four categories GS, BR, GR, and BS.

The numbers of nonconforming items (in parts per thousand) that will be shipped by regular inspection (using the customer specifications) in each of the different situations are shown in Figure 6. (These values are the sums of the ppt for the regions BS1 and BS2 in Figure 5.)

ppt bad shipped		ICC	ICC	ICC	ICC	ICC	ICC	ICC	ICC	ICC
	ppt bad	0.995	0.99	0.98	0.96	0.94	0.92	0.90	0.88	0.86
C_p	produced									
1.00	3 ppt	0	0	0	0	1	1	1	1	1
0.90	7 ppt	0	1	1	1	1	1	2	2	2
0.85	11 ppt	1	1	1	2	2	2	2	3	3
0.80	16 ppt	1	1	2	3	3	3	4	4	4
0.75	24 ppt	1	2	3	4	4	5	5	6	6
0.70	35 ppt	2	3	4	5	6	7	7	8	8
0.65	51 ppt	3	4	5	7	8	10	10	11	12
0.60	71 ppt	3	5	7	10	11	13	14	15	16
0.55	98 ppt	4	6	9	13	15	17	19	20	21
0.50	132 ppt	6	8	12	16	20	22	24	26	28
0.45	175 ppt	7	10	15	21	25	28	31	33	35
0.40	228 ppt	9	13	18	25	31	35	38	41	44
0.35	291 ppt	10	15	22	30	37	42	47	50	54
0.30	365 ppt	12	18	25	36	44	50	55	60	64
0.25	450 ppt	14	20	29	41	50	58	64	70	75
0.20	545 ppt	15	23	33	47	57	66	73	80	86

ppt bad shipped		ICC	ICC	ICC	ICC	ICC	ICC	ICC	ICC	ICC
	ppt bad	0.84	0.82	0.80	0.75	0.70	0.60	0.50	0.40	0.30
C_p	produced									
1.00	3 ppt	1	1	1	1	1	1	1	1	1
0.90	7 ppt	2	2	2	2	2	2	3	3	3
0.85	11 ppt	3	3	3	3	3	4	4	4	4
0.80	16 ppt	4	4	5	5	5	6	6	6	7
0.75	24 ppt	6	6	7	7	8	8	9	9	10
0.70	35 ppt	9	9	10	10	11	12	13	14	14
0.65	51 ppt	12	13	13	14	15	17	18	19	20
0.60	71 ppt	17	18	18	20	21	23	25	27	28
0.55	98 ppt	22	23	24	27	28	31	34	36	38
0.50	132 ppt	29	31	32	35	37	41	45	48	50
0.45	175 ppt	37	39	41	45	48	54	58	62	63
0.40	228 ppt	47	49	51	56	61	68	74	78	79
0.35	291 ppt	57	60	63	69	75	84	91	95	94
0.30	365 ppt	68	72	76	83	90	101	108	111	108
0.25	450 ppt	80	85	89	98	106	117	123	124	118
0.20	545 ppt	91	97	101	111	119	129	132	130	121

Figure 6: Parts per Thousand Nonconforming Shipped by 100% Inspection Using Customer Specs

The first four entries in the first row and the first entry in the second row show where regular 100% inspection will ship zero parts per thousand nonconforming. All you need is a virtually perfect measurement system and a process that produces fewer than 10 ppt nonconforming.

The remaining 283 entries show the effects of imperfections in the measurement system. The lower the intraclass correlation coefficient the greater the number of nonconforming items that will pass inspection. The lower the capability ratio the greater the number of nonconforming items produced, and thus the greater the number that get shipped.

When we compare the number of nonconforming items produced with the number shipped we can see that regular inspection will screen out substantial proportions of the nonconforming items. However, it will rarely be perfect. So, if you are depending upon 100% inspection for quality assurance you will need to be ready to live with imperfection.

MANUFACTURING SPECIFICATIONS

One of the attempts to remedy the imperfection of inspection using the customer specifications (regular inspection) is to use tightened inspection. Guard-bands are placed on the conforming side of the customer specifications and inspection is performed using these tightened manufacturing specifications.

For over 30 years this author, along with others, has suggested the use of the median error of a measurement, known as the Probable Error (PE), as the basic unit for establishing these guard-bands. We use the probable error as the basic increment for guard-bands because it defines the effective resolution of the measurement system. Half the time a measurement will err by one PE or more, and half the it will err by one PE or less. For this reason, trying to use smaller increments for guard-bands does not make sense.

(The probable error of a measurement is 0.675 times the standard deviation of repeated measurements of the same thing. It can be computed whenever a measurement system evaluation has been carried out.)

The customer specifications might be tightened by one, two, three, or four probable errors and used as manufacturing specifications. The bivariate normal models were evaluated for all four categories (GS, BR, GR, and BS) for four different sets of manufacturing specifications (guard-bands of one-PE, two-PE, three-PE, and four-PE). This was done for all 288 situations defined in Figure 6.

Since the use of manufacturing specifications implies a desire to capture more nonconforming product, we have to conclude that the motivation behind manufacturing specifications is an attempt to ship zero nonconforming product. It turns out that guard-bands can be defined that will achieve zero parts-per-thousand shipped for 168 of the 288 situations considered.

Of course, at the same time that the manufacturing specifications capture more nonconforming items (to reduce BS), they will also reject more conforming items (to increase GR) as shown in Figure 7. This reduces the number of good items shipped and effectively increases the unit cost of the shipped items.

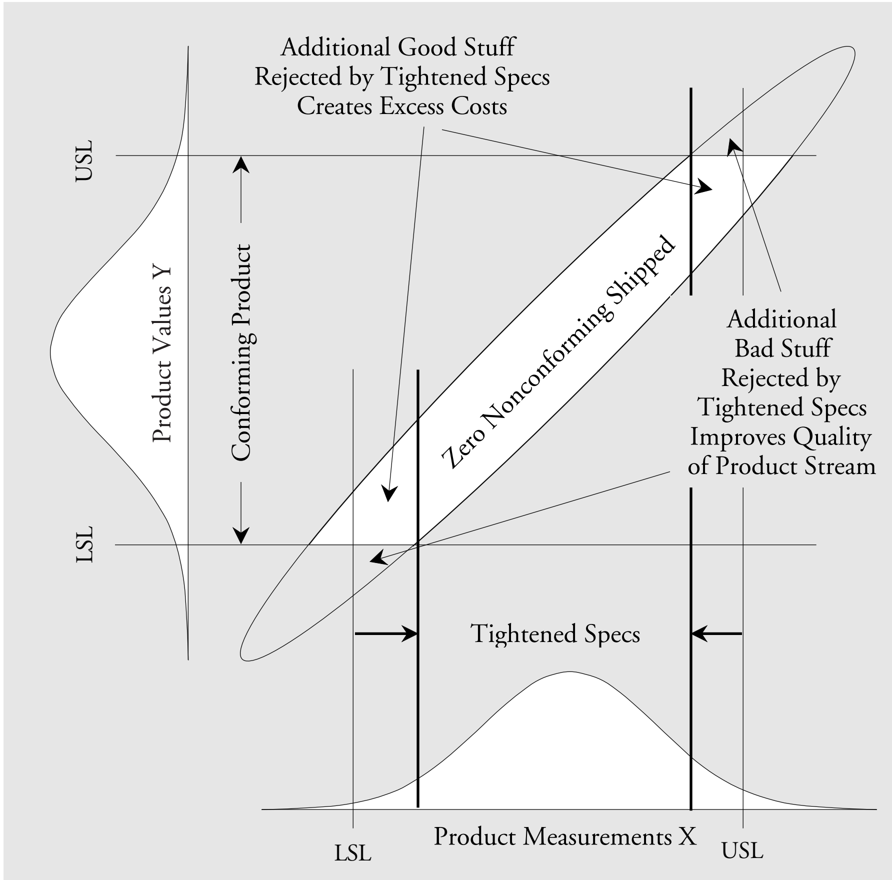

Figure 7: The Effect of Guard-bands

The excess cost of tightened inspection is quantified by the percentage by which the unit cost of the shipped items is increased relative to the unit cost resulting from regular inspection. These percentages will be used in the following tables to characterize the zero ppt nonconforming inspection plans.

Figure 6 quantifies the imperfections of regular inspection by listing the ppt nonconforming items that remain. In order for a tightened inspection plan to be a zero ppt nonconforming plan it will have to capture *all* of these residual nonconforming items. Thus, the numbers in Figure 6 define the benefit derived from any zero ppt nonconforming inspection plan. These numbers are listed right below the excess costs for the plans in the following tables.

ONE PROBABLE ERROR GUARD-BANDS

We start with manufacturing specifications that are tightened by one probable error on each side. Guard-bands of one probable error on each side will result in zero ppt nonconforming inspection plans for the twenty configurations shown in Figure 8. In the following tables two numbers are given. The first, expressed as a percentage, is the excess cost. The second, right below the first, is the parts per thousand nonconforming removed from the shipped product stream by that inspection plan.

1 PE Plans C_p	ICC 0.995	ICC 0.99	ICC 0.98	ICC 0.96	ICC 0.94	ICC 0.92	ICC 0.90	ICC 0.88
1.00					0.2% 1	0.3% 1	0.4% 1	0.5% 1
0.90		0.1% 1	0.2% 1	0.3% 1				
0.85	0.1% 1	0.2% 1	0.3% 1					
0.80	0.2% 1	0.3% 1						
0.75	0.3% 1							

Excess Cost Incurred
ppt nonconforming removed

1 PE Plans C_p	ICC 0.86	ICC 0.84	ICC 0.82	ICC 0.80	ICC 0.75	ICC 0.70	ICC 0.60
1.00	0.6% 1	0.7% 1	0.8% 1	1.0% 1	1.4% 1	2.0% 1	4.0% 1

Figure 8: One Probable Error Zero ppt Nonconforming Plans

In these 20 situations, where regular inspection will ship one part-per-thousand nonconforming, the use of manufacturing specifications with one-PE guard-bands will allow you to ship zero ppt nonconforming! The excess costs for these one-PE plans vary from 0.1% to 4.0% depending upon the situation. It is only when the *ICC* value is larger than 0.95 that these plans will work with capabilities that are less than 1.00. Thus, one-PE guard-bands will help in those situations where regular inspection is already almost perfect.

TWO PROBABLE ERROR GUARD-BANDS

Manufacturing specifications that are tightened by two probable errors on each side will result in zero ppt nonconforming plans for the 41 situations shown in Figure 9.

2 PE Plans C_p	ICC 0.995	ICC 0.99	ICC 0.98	ICC 0.96	ICC 0.94	ICC 0.92	ICC 0.90	ICC 0.88	ICC 0.86
0.90					1.3% 1	1.7% 2	2.2% 2	2.8% 2	3.3% 2
0.85				1.3% 2	1.8% 2	2.4% 2	3.0% 2	3.8% 3	4.5% 3
0.80			1.0% 2	1.8% 3	2.5% 3	3.3% 3	4.1% 4	5.1% 4	
0.75		0.9% 2	1.4% 3	2.4% 4					
0.70	0.9% 2	1.3% 3	1.9% 4						
0.65	1.2% 3	1.7% 4							
0.60	1.6% 4	2.3% 5							
0.55	2.1% 5								

Excess Cost Incurred
ppt nonconforming removed

2 PE Plans C_p	ICC 0.84	ICC 0.82	ICC 0.80	ICC 0.75	ICC 0.70	ICC 0.60	ICC 0.50	ICC 0.400	ICC 0.300
1.00							27.8% 1	55.5% 1	129.5% 1
0.90	4.1% 2	4.8% 2	5.6% 2	8.4% 2	11.8% 2	22.7% 3			
0.85	5.5% 3	6.4% 3	7.5% 3	11.0% 3					

Figure 9: Two Probable Error Zero ppt Nonconforming Plans

In these 41 situations, where regular inspection will ship from 1 to 5 parts-per-thousand nonconforming, the use of manufacturing specifications with two-PE guard-bands will allow you to ship zero ppt nonconforming. These 41 two-PE plans have excess costs that vary from 1% to 130%. Clearly only the plans with the larger *ICC* values even begin to be feasible here—excess costs of 1% to 3% might occasionally be acceptable to remove the last 0.2% to 0.5% nonconforming. Before using two-PE guard-bands you should consider if the improvement in the shipped product stream is worth the excess cost involved.

THREE PROBABLE ERROR GUARD-BANDS

Manufacturing specifications that are tightened by three probable errors on each side will result in zero ppt nonconforming plans for the 60 configurations shown in Figure 10.

3 PE Plans C_p	ICC 0.995	ICC 0.99	ICC 0.98	ICC 0.96	ICC 0.94	ICC 0.92	ICC 0.90	ICC 0.88
0.75					6.5% 4	8.9% 5	11.2% 5	14.3% 6
0.70				5.9% 5	8.7% 6	11.8% 7	14.8% 8	18.9% 8
0.65			4.7% 6	7.9% 8	11.6% 9	15.6% 10	19.6% 11	24.9% 12
0.60			6.2% 8	10.5% 10	15.2% 12	20.6% 14		
0.55		5.0% 7	8.2% 10	13.8% 14	20.0% 17			
0.50	4.3% 7	6.6% 10	10.8% 14	18.1% 19				
0.45	5.5% 9	8.5% 13	14.0% 18					
0.40	7.2% 11	11.1% 17						
0.35	9.4% 15	14.6% 22						
0.30	12.3% 19							

Excess Cost Incurred
ppt Nonconforming Removed

3 PE Plans C_p	ICC 0.86	ICC 0.84	ICC 0.82	ICC 0.80	ICC 0.75	ICC 0.70	ICC 0.60	ICC 0.50
0.90								156.5% 3
0.85						39.5% 4	85.2% 4	219.3% 4
0.80	13.3% 4	15.9% 4	19.3% 5	22.8% 5	34.6% 5	51.5% 5	113.5% 6	
0.75	17.6% 6	20.9% 6	25.4% 7	29.9% 7	45.2% 8	67.7% 8		
0.70	23.2% 9	27.5% 9	33.3% 10	39.2% 10	59.6% 11			
0.65	30.5% 13	36.1% 13	43.8% 14					

Figure 10: Three Probable Error Zero ppt Nonconforming Plans

In these 60 situations, where regular inspection will ship from 3 to 22 parts-per-thousand nonconforming, the use of manufacturing specifications with three-PE guard-bands will allow you to ship zero ppt nonconforming. These 60 three-PE plans have excess costs that vary from 4% to 219%. Once again, only those situations with the larger *ICC* values are even remotely feasible. For the 20 situations where *ICC* exceeds 0.95 the average excess cost for each single ppt nonconforming removed is 0.78%. Thus, on the average, the costs go up here about eight times faster than the number of nonconforming items removed.

FOUR PROBABLE ERROR GUARD-BANDS

Manufacturing specifications that are tightened by four probable errors on each side will result in zero ppt nonconforming plans for the 47 situations shown in Figure 11.

4 PE Plans C_p	ICC 0.995	ICC 0.99	ICC 0.98	ICC 0.96	ICC 0.94	ICC 0.92	ICC 0.90
0.60	Excess Cost Incurred ppt Nonconforming Removed						48.5% 15
0.55						49.1% 19	65.8% 21
0.50					46.1% 23	67.0% 26	91.7% 29
0.45				40.4% 25	62.9% 31	94.3% 35	134.6% 39
0.40			29.7% 24	55.1% 33	89.1% 41	141.8% 46	219.9% 52
0.35			40.2% 31	78.3% 44	135.9% 54		
0.30		30.7% 28	56.5% 41	120.6% 58	245.9% 71		
0.25	25.9% 25	43.4% 37	85.7% 54				
0.20	37.7% 34	67.1% 51	156.0% 74				

4 PE Plans C_p	ICC 0.88	ICC 0.86	ICC 0.84	ICC 0.82	ICC 0.80	ICC 0.75	ICC 0.70
0.70							312.1% 12
0.65					123.2% 15	242.1% 16	657.1% 17
0.60	64.0% 17	82.4% 18	106.9% 19	140.3% 20	182.3% 20	433.6% 22	
0.55	88.5% 23	116.6% 24	156.4% 26	216.2% 27	301.1% 28		
0.50	127.4% 31	175.6% 33	252.3% 35				
0.45	199.4% 42						

Figure 11: Four Probable Error Zero ppt Nonconforming Plans

While these plans provide the opportunity for substantial improvements to the product stream, they do so at a very high cost. Ten of these plans with *ICC* of 0.90 or larger have excess costs below 50% and will remove the last 15 to 37 ppt nonconforming remaining. Increasing the overall cost by 26% to 50% in order to remove the last 1.5% to 3.7% of nonconforming product is a very steep price to pay. These high excess costs make four-PE guard-bands impractical for all but the most critical of components.

FEASIBLE GUARD-BAND PLANS

Clearly guard-bands are feasible only when the measurement system is good enough to have a large *ICC*. In Figures 8, 9, and 10 there are 46 plans with large *ICC* values (≥ 0.92) and excess costs below 10%. These 46 plans define situations where guard-bands may occasionally be feasible. However, since only five of these 46 plans improve the shipped quality by 1% (10 ppt) or more, we have to concede that feasible guard-band plans will not have a large impact on the outgoing quality.

So, tightened inspection will improve the quality of the shipped product stream, but it will also create excess costs. In the 1152 cases considered in preparing this paper, the marginal excess cost of tightened inspection always increased faster than the quality level improved. When you can live with some nonconforming product being shipped, stick with regular inspection. The marginal costs of using tightened inspection are so high that anything less than zero defects simply cannot justify their use.

HOW TO AVOID SHIPPING NONCONFORMING PRODUCT

The best way to avoid shipping nonconforming product is to avoid making any bad stuff in the first place.

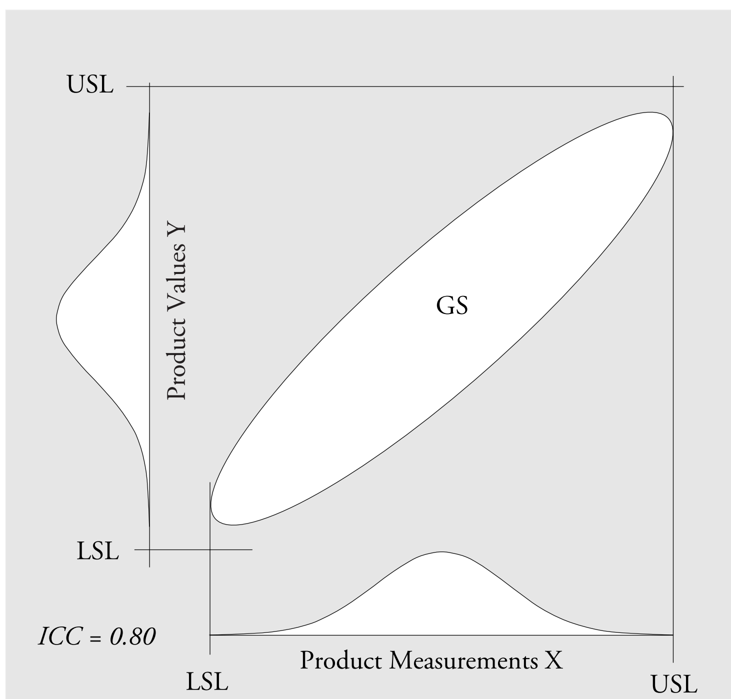

Figure 13: The Only Sure Way to Avoid Shipping Nonconforming Product

In order for the three-sigma ellipse to fit entirely within the *GS* region we will need to operate our process predictably, on-target, and with a capability ratio comfortably in excess of 1.00. Here we will find 100% inspection to be unnecessary. This will not only save the cost of the perfect measurement system needed for effective inspection, but it will also eliminate the costs of inspection and thereby increase productivity.

But how can you do this? A proven way of reducing both process variation and production costs is through the effective use of process behavior charts for continual improvement. Moreover, process behavior charts can be effective even when the measurement system is less than perfect. They will work in virtually all of the 288 situations listed in Figure 6.

Improving the production process will reduce both the excess costs of production and the excess costs of use. Moreover, as has been proven time and again, these process improvements can usually be accomplished without capital expenditures and without having to improve the imperfect measurement system. It is always better to learn how to quit burning the toast than to be a world-class toast scraper!

SUMMARY

If you are wedded to using inspection at the end of the line to separate the good stuff from the bad stuff, then you will need to be committed to using only perfect measurement systems. Of course, when virtually perfect measurement systems exist, they will tend to be expensive to own and operate. Since money spent to purchase and operate measurement systems is 100% overhead, these costs directly reduce productivity and profitability. They do not make the product any better. They do not eliminate the need for inspection. They simply increase the cost of scraping the burnt toast.

On the other hand, less than perfect measurement systems may be used with process behavior charts to substantially improve the quality and consistency of a production process. Time after time my students report two-fold, three-fold, and even four-fold improvements in process capability. These improvements can eliminate the need for 100% inspection while they reduce other excess costs for both the producer and the customer. They allow you to quit burning the toast.

The choice is yours. The excess costs of tightened inspection and buying better measurement systems will increase your overhead, lower your productivity, and hurt your competitive position. Using your imperfect measurement systems with process behavior charts to improve your production process capability will allow you to eliminate the need for inspection, increase your productivity, and improve your competitive position.